The present paper attempts to examine the main ideas of Sjaastad (1998) paper and criticize them in the light of a more practical approach that can be used in real world economies. In general Sjaastad (1998) paper focuses on clarifying why the so called Purchasing Power Parity Theory (PPP) is a very bad measure of the true Real Exchange Rate (RER), which according to his definition should be the ratio of the price index for traded goods to nontraded goods.

In order to show that PPP is a bad proxy of the real exchange rate Sjaastad uses a measure that attempts to capture the difference between the RER based on PPP and what he defines as the true RER. This measure is represented by the fraction of the variance of the RER based on PPP that has no counter-part in the variance of the true real exchange rate. However it seems to me that in particular, the author focuses his study on showing that the PPP approach to RER is a bad proxy of the true real exchange rate, since the first one is a bilateral concept, and the ratio of the tradables to non-tradables is a multilateral concept since it takes into account multilateral trade variables.

In the first part of this paper, I present what I consider the main ideas of the Sjaastad and next I will present the critique to his 1998 paper using as tools of analysis some Harberger’s concepts on RER (Harberger 1988). In particular, an evaluation of different RER definitions based on economic theory is presented, to see if those definitions satisfy the requirements needed to be a useful tool of analysis at theoretical and empirical level.

One of the most important issues that will be concluded in this paper is that a good definition of the exchange rate must satisfy not only statistical but mainly economic conditions. Another key point in this paper will be to emphasize that the alternative expression of the RER as the ratio of price index of tradables to non-tradables, which was derived by Sjaastad could work better than other multilateral RER definitions which will be analyzed later on.

Sjaastad’s Properties of Real Exchange Rate
(Some Definitions)
The following basic notation and definitions will be necessary to develop our analysis on RER; the notation is taken from

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Sjaastad, and upper case letters indicate natural logarithms of the correspondent variables.

\[ EX^y = \text{price of currency } Y \text{ in terms of currency } X. \]

\[ PX^x (PM^x) = \text{a price index of country } X\text{'s exports (imports)} \]

\[ PT^x (PH^x) = \text{a price index of country } X\text{'s traded (nontraded) goods} \]

\[ P^x = w PH^x + (1-w_y)PT^x = \text{an over all price index for country } x \]

\[ PREP^x = \text{the PPP real exchange rate of country } x, \]

\[ TRER^x = \text{the true RER of country } X \]

defined as the ratio of tradables to non-tradables.

\[ TT^x = PX^x - PM^x = \text{terms of trade} \]

Sjaastad defines the true real exchange rate as but because is not easy to measure he prefers the definition

\[ TRER^x = PT^x - P^x = w PH^x + (1-w_y)PT^x \] \hspace{1cm} (1)

which differs from the original definition above in the factor of proportionality \( w \).

The PPP real exchange rate for country \( X \) vs a vis country \( Y \) is defined as

\[ PRER^y = P^y + EX^y - P^x = PF^y - P^x \] \hspace{1cm} (2)

where \( PF^y \) is the countries \( Y \)’s price level denominated in countries \( X \)’s currency.

The definition of tradables price index is the following:

\[ PT^x = \sum_{j=1}^{M} \theta_j^x PF_j^y + G(Z^x) \] \hspace{1cm} (3)

Where the term \( G(Z) \) reflects the effects of all other variables (the fundamentals) and measures the relative market power of country \( j \) over the prices of country \( X\)’s traded goods. Then, if we neglect the term \( G(Z) \) we get:

\[ TRER^x = PT^x - P^x = \sum_{j=1}^{M} \theta_j^x (PF_j^y - P^x) = \sum_{j=k}^{M} \theta_j^x PRER_j^x \] \hspace{1cm} (4)

Combining now this last expression with the identity we have:

\[ PRER^y = PRER^y + PRER^y \]

which implies:

\[ TRER^x = \sum_{j=k}^{M} \theta_j^x PRER_j^x - (1 - \theta) PRER^y \] \hspace{1cm} (4’)

Then the RER based on PPP and the TRER will be co-linear if

\[ \sum_{j=k}^{M} \theta_j^x PRER_j^x \]

is a constant, or if \( \theta_j^x = \theta \forall j \neq x, y \). Notice that the first condition will be satisfied only if purchasing power parity holds perfectly between country \( Y \) and all other countries. The second condition implies that \( \theta_j^x + \theta_j^y = 1 \); that is, country \( X \) and \( Y \) taken together must be price makers in the world markets for country \( X\)’s traded goods. In the next section, the definition of the error in the mea-
sure of PPP approach to RER will be presented.

**Error Measurement for the Real Exchange Rate based on PPP.**

This part can be considered as the estimation procedure for the measure of divergence between the RER based on PPP and the TRER. To develop this measure we start with the definitions of both RER concepts given above. Combining

\[ \text{PRER}^y_x = P^y_y + EX - P^x_x \]

with

\[ \text{TRER}^x_x = PT^x_x - P^x_x \]

we get:

\[ \text{PRER}^y_x = \text{TRER}^x_x + E^y_x \] (5)

where \( E^y_x = P^y_y - PT^x_y \), which can be considered as the deviation of the RER based on PPP from TRER.

Now, following Sjaastad it is necessary to calculate the variance of the RER based on PPP that has no counterpart in the TRER and in order to do so we need to compute the variance of equation (5). This can be re-expressed as:

\[ \sigma_{P,E}^2 = \sigma_{P}^2 - \sigma_{P,T}^2 \]

where \( \sigma_{P}^2 \) is the variance of the RER based on PPP, \( \sigma_{P,E} \) is the covariance between RER based on PPP and \( E^y_x \), and finally, \( \sigma_{P,T} \) is the covariance of RER based on PPP and \( \text{TRER}^x_x \).

In order to estimate the variance \( \sigma_{P,E}^2 \) we can decompose equation (5) in the following two equations

\[ \text{PRER}^y_x = \alpha E^y_x + u^y_x \] (6)

\[ \text{TRER}^x_x = (\alpha-1)E^y_x + u^x_x \] (7)

The bilateral PPP exchange rate is defined as:

\[ e = E \frac{P^*}{P_d} \]

where \( E \) is the nominal exchange rate (units of domestic currency per unit of foreign currency), \( P^* \) is the price index of a particular foreign country (expressed in foreign currency), and \( P_d \) is a price index of the domestic country. Notice that under this definition \( e \) represents the real price of a real unit of foreign currency. We obtain the real price of one unit of foreign currency from dividing the nominal exchange rate \( E \) by a domestic price index and then we get one unit of foreign currency at constant purchasing power if we divided our last result by some foreign price index. This definition will work well (satisfying our definition of RER) if world trade were a bilateral issue, or if one country and only one country were price maker in the world trade. But if the world trade is multilateral then the PPP is very likely to do a very bad job in satisfying our conditions: to equilibrate the real demand and supply of foreign exchange. Because actually the trade among countries is multilateral and not restricted only to bilateral trade activities, we will have that for some countries the PPP RER will generate an excess supply of foreign currency in real terms. On the other hand, for some other
countries the PPP RER will imply to have an excess demand for foreign currency in real terms. In other words, for some countries this definition of the RER will imply to have an overvalued RER and for others will imply to have an undervalued RER.

Our next Real Exchange Rate definition is called by Sjaastad the True Real Exchange Rate, which is expressed as the ratio of the price indexes of tradables to nontradables goods. In my point of view this is a good definition of the RER in some sense, because it avoids the problem that RER based on PPP suffers from, which is to define the real exchange rate using as foreign price index a weighted average of the price indexes of import an export goods, or as equation (4) expresses, a weighted average of the RERs based on PPP of M economies using as weight an index of market power possessed by country j over the price of country X’s traded goods.

Whose residuals are identical, and OLS estimates of those residuals \( \hat{u}_x \) are also identical. Since \( \hat{u}_x \) is orthogonal to \( E_x \) in equation (6), the variance of those residuals constitutes the clean portion of the variance of the PPP real exchange rate. Finally the relative variance is obtained by running the regression \( \hat{u} = constant + \beta PREP^x \) and the estimate of the relative error is \( 1 - \beta \).

The next step in this analysis is to describe the theory that will help us to criticize Sjaastad’s paper. We need to start by emphasizing the fact that, in any country, the real exchange rate (RER) is a very important policy variable, and as Sjaastad paper establishes, measuring it with accuracy is of a great importance. However, accuracy in economics may have not a univocal meaning. Hence, when we present the RER definition later on, we will diverge from Sjaastad accuracy’s point of view. In my opinion accuracy of an economic measure, in particular accuracy of a price, will depend on how well the price reflects what is happening in the market (quantities supplied and demanded) of the good in question. Therefore, it can be said that price fluctuation of any good could be caused by fluctuations in the quantities, and this does not necessarily mean that “our price” is a bad measure of the “true” price, but that the measure is precisely capturing what is going on in the market.

The previous argument can be used to help us find a good definition of the Real Exchange Rate. In general, we can say that the RER is a real price (relative price) that makes the real demand and real supply for foreign currency to be in equilibrium. From this it can be said that we have a good definition if our RER tell us the appropriate magnitude and direction of the change in RER when an excess of demand or supply for foreign exchange is present.

Now let us analyze three different approaches or definitions to the RER: the PPP (bilateral) definition of exchange rate, the ratio of tradables to nontradables definition, and a multilateral version of the PPP real exchange rate.

The definition of the RER as the price ratio of the tradables to non-tradables works well in the sense that it sends the right signal to the market in order to allocate the resources in the right sector (tradables or non-tradables), and it also works very well in equilibrating the foreign exchange market in almost all of the cases, however as Harberger (1988) points out “the price ratio of tradables to nontradables is not particularly helpful as a key to critical
insights or as analytical concept or tool when we have differential movements in the price of tradables or different forces influencing production separate one or more tradables from the rest. Examples include a world oil boom, looked at either from the standpoint of an oil-importing country; a reduction in real costs of producing a particular tradables good, either locally or worldwide; and an the introduction or relaxation of trade restrictions”.

As a conclusion to the previous paragraph it can be said that the low relative variance of any price is not enough to be a good signal for resource allocation. Additionally, the RER measured as the ratio of the price index of tradables to non-tradables will give us, in some cases, a wrong signal for the resource allocation, property that makes it not a good definition of RER, and a useless measure for policy design, just like the RER based on PPP. At this point, I want to say that the transformation of the price ratio of tradables to non-tradables presented by Sjaastad in equation (4), could avoid (very well) the pitfalls of the price ratio that Harberger have pointed out, however Sjaastad did no use this transformation in his estimations.

In order to clarify this last paragraph I would like to present now a RER definition that fulfills the requirements that we impose over a good definition of “the real price of foreign exchange”. The definition of the RER that I want to analyze now is the so called multilateral PPP. This definition of RER is considered by some authors as one that will capture any kind of movement in the supply and demand for foreign exchange, and with this characteristic, the multilateral PPP satisfies the condition we need for the RER measure which is to be a real price for a real dollar. The definition of the multilateral PPP is given by \( e = E\left(\frac{P^*}{P_d}\right) \), and it is important to highlight now that \( P^* \) is not a price index of a specific foreign country, but under this new multilateral approach, \( P^* \) represents a general index of the world market prices, and \( P_d \) continues representing a price index of the country in question. In particular, for the multilateral approach, the \( P^* \) index should represent a weighted average of foreign prices, or equivalently prices of a representative basket of tradables goods from all around the world.

As we can notice, the multilateral PPP differs from the bilateral PPP just in the definition of the term \( P^* \), then in order to avoid the pitfalls in the last one, we need to make sure that \( P^* \) would be the same deflator used to describe the real supply and real demand for foreign exchange. Further more if we choose \( P^* \) correctly, it must allows us that \( EP^*/P_d \) will equilibrate the real foreign market. Harberger have proposed to get \( P^* \) as a weighted sum of the wholesale prices index of the major trading countries, using as weights the SDR weights and the WPI of the USA, Germany, France Japan, and U.K. I would like to mention at this point that, under my perception, it seems that Sjaastads equation number (4) is very similar to the Harberger’s definition of the multilateral PPP, but the first one has more theoretical support in the sense that using the country’s market power as weighting factor we are giving to each foreign \( P^* \) (in the multilateral PPP) its true relevance in the world market, the SDR weights however, could be arbitrary and not necessarily with a heavy theoretical support. Despite this, it is necessary to mention that, in the
estimation of the RER Sjaastad did not used his equation (4) and he gives preference to the RER definition based on the ratio of tradables to nontradables indexes when working on the estimation and uses this definition to show that the variance of the bilateral PPP is higher that variance of the RER based on the ratio of tradables to nontradables. But as we have seen, in reality none of them necessarily reflects what is happening in the market of real demand and supply of foreign currency, and additionally, knowing if the variance is low or high, helps nothing when the task is the designing of exchange rate policy.

In conclusion I can say that Sjaastad’s paper in estimating and showing that the variance of the bilateral RER based on PPP is consistently higher than the variance of the RER based on the price ratio of tradables to nontradables, is not enough to show that the last one is the right measure of the real price of the real dollar, which is the correct economic meaning of the RER variable. It could happen that the variance in some price variable is explained by the market circumstances and not necessarily because the price measure is a bad measure “per se”. Consequently, in my opinion what Sjaastad’s paper (1998) proves is that the bilateral RER based on PPP and the RER based on tradables to nontradables ratio, are not the same thing and statistically they will differ in the most of the cases.

BIBLIOGRAPHY

